Computing Temperature Field within a Melt Spinline

In a previous paper[1] the author showed that the temperature field within a molten polymer filament while in air-cooled steady-state melt spinning is equivalent to a two-dimensional transient heat conduction within a circular disk of fixed radius $p = 1$ governed by the equation

$$\frac{\partial t}{\partial X} = \frac{1}{p}\frac{\partial t}{\partial p} + \frac{\partial^2 t}{\partial p^2} + \frac{1}{p^2}\frac{\partial^2 t}{\partial \theta^2} \tag{1}$$

where t is temperature, X is a time like variable, and p and θ are the radius and angle of the cylindrical coordinates. Further, in this heat conduction problem, temperature t throughout the disk is initially made equal to the spinneret temperature. Thereafter, temperature outside of the disk is made to suddenly lower to the cooling air temperature, and the heat transfer coefficient at the disk periphery is made to change with angle θ and time X in a certain predetermined manner.

In solving the above heat conduction problem numerically, eq. (1) was approximated by dividing the circular disk into 81 elements as shown in Figure 1, taking each element to be of uniform temperature and taking the coefficient of heat transfer between each element equal to the inverse of the distance between centers of two neighboring elements.

In the present note the author proves that the above division of the disk into elements as shown in Figure 1 is a proper approximation to the partial differential equation (1). This proof happened to be lacking in the previous paper.[1] If we take a first-order forward differencing scheme to the derivative $\partial/\partial X$ and a central differencing scheme to the derivatives $\partial/\partial p$, $\partial^2/\partial p^2$, and $\partial^2/\partial \theta^2$ difference equation below is obtained (see Fig. 2):

$$\frac{t_{i,j}^{\text{new}} - t_{i,j}}{\Delta X} = \frac{t_{i+1,j} - t_{i-1,j}}{2p_i\Delta p} + \frac{t_{i+1,j} - 2t_{i,j} + t_{i-1,j}}{\Delta p^2} + \frac{t_{i,j+1} - 2t_{i,j} + t_{i,j-1}}{p^2\Delta\theta^2} \tag{2}$$

The above eq. (2) take the form below when rearranged with the arc length $p\Delta\theta$ replaced by $\Delta q = p\Delta\theta$:

$$(\Delta p\cdot\Delta q)\,\frac{t_{i,j}^{\text{new}} - t_{i,j}}{\Delta X} = \Delta q\left(1 + \frac{\Delta p}{2p_i}\right)\frac{t_{i+1,j} - t_{i,j}}{\Delta p} + \Delta q\left(1 - \frac{\Delta p}{2p_i}\right)\frac{t_{i-1,j} - t_{i,j}}{\Delta p} + \Delta p\,\frac{t_{i,j+1} - t_{i,j}}{\Delta q} + \Delta p\,\frac{t_{i,j-1} - t_{i,j}}{\Delta q} \tag{3}$$

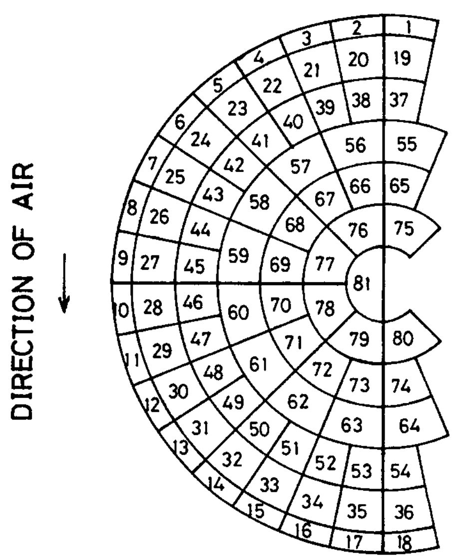

Fig. 1. Filament cross section divided into 81 elements.[1]

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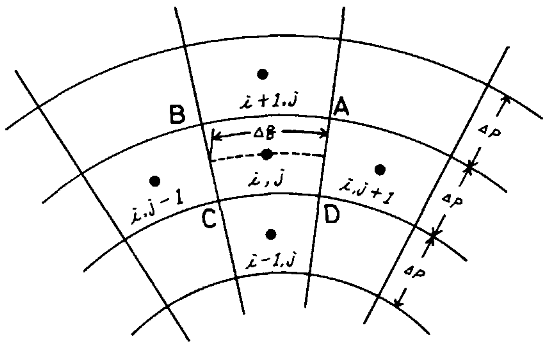

Fig. 2. Differencing in cylindrical coordinates.

Equation (3) above is nothing other than the heat balance formula for the center element (i, j) in Figure 2 exchanging heat with four adjacent elements through the four walls AB, BC, CD, and DA. In eq. (3) $\Delta p \Delta q$ gives the volume of the center element (i, j) assuming a unity thickness of the disk, $\Delta q(1 + \Delta p/2p_i)$ is the arc length AB, Δp is the wall length DA = BC, and $\Delta q(1 - \Delta p/2p_i)$ is the arc length CD. Heat transfer coefficient at walls AB and CD is equal to the inverse of radial center distance Δp and that at walls DA and BC equals the inverse of transverse center distance Δq. Equation (18) of the previous paper[1] used in numerical computation of temperature field can be readily derived from eq. (3) above.

The division of the circular disk into numerous elements as shown in Figure 1, therefore, is a proper approximation of the partial differential equation (1). The fact that both $\Delta\theta$ and Δp varies with radial distance p does not affect the validity of the approximation as far as each element is sufficiently small and remains bounded by lines of constant angle and arcs of constant radius.

References

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